Feast Fest
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Can viscosity turn inflation into the CMB and Lambda?

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The end of inflation occurs as the CMB is created. In this period there is both an inflationary density and a radiation density each with its own pressure

$$p_i = -\rho_i c^2, \qquad p_r = (1/3)\rho_r c^2$$

$$p = p_i + p_r \qquad \rho = \rho_i + \rho_r$$

Kumar Chitre and I argue that there will be
a bulk viscosity even though
radiation alone has no bulk viscosity,
and inflationary density is assumed to
have no bulk viscosity.

Why do we say this?

We argue by analogy with two inviscid gases
with different adiabatic indexes gamma.
Let them be at one temperature initially;
then after expansion the one with the greater
index will be cooler so heat will flow
and entropy will be generated.
Thus expansion or contraction will create
entropy; that is described in fluid mechanics
by a bulk viscosity zeta.
Evidently such a zeta must vanish if only
one gas is present.

For the two gases the entropy generation occurs due to the exchange of energy between them caused by the expansion.

Now consider the different system of inflationary density and the CMB. The energy of inflation is transferred and that creates the entropy of the CMB. We claim this is all done through an effective bulk viscosity.

The second law is then

$$TdS/dt = d/dt(\rho c^2 V) + pdV/dt = \zeta \dot{V}^2/V \quad (1)$$

The energy of inflation decreases
as it is transferred to the CMB via the bulk viscosity

$$V d(\rho_i c^2)/dt = -\zeta \dot{V}^2/V \qquad (2)$$

Einstein's equation for the scale factor of cosmology is not affected by bulk viscosity

$$\frac{\dot{a}^2 + kc^2}{a^2} = (8\pi/3)G\rho, \quad k = \pm 1, 0 \qquad *$$

$$V = (4/3)\pi a^3$$

near the end of inflation curvature is negligible so

$$9\frac{\dot{a}^2}{a^2} = [\frac{\dot{V}}{V}]^2 \doteq 24\pi G\rho = \omega^2 X^2 \qquad (3) \quad *$$

$$X^2 = \rho/\rho_0, \quad \rho_0 = const \quad \omega^2 = 24\pi G\rho_0$$

Inflationary density is Lorentz invariant so it does not have a rest frame or a velocity, so we use the CMB to define the velocity and the rest frame for the bulk viscosity, which must be zero if only one component exists.

$$\zeta = \nu_0 \rho_r \sqrt{\rho_i/\rho}$$

this vanishes appropriately.

Here ν_0 is a kinematic bulk viscosity of dimensions L^2/T, and the square root makes the maths easier. We use $\nu = (3/4)\nu_0 \omega/c^2$, which is dimensionless.

We divide equation (1) by equation (2) and with $Y^2 = \frac{\rho_i}{\rho_0}$, $X^2 = \frac{\rho}{\rho_0}$, we find

$$\frac{XdX}{YdY} = \frac{\omega XV}{\nu Y\dot{V}} - 1 \qquad (\nu^{-1} - Y)\frac{dY}{dX} = X \qquad *$$

$$X^2 = (2/\nu)Y - Y^2$$

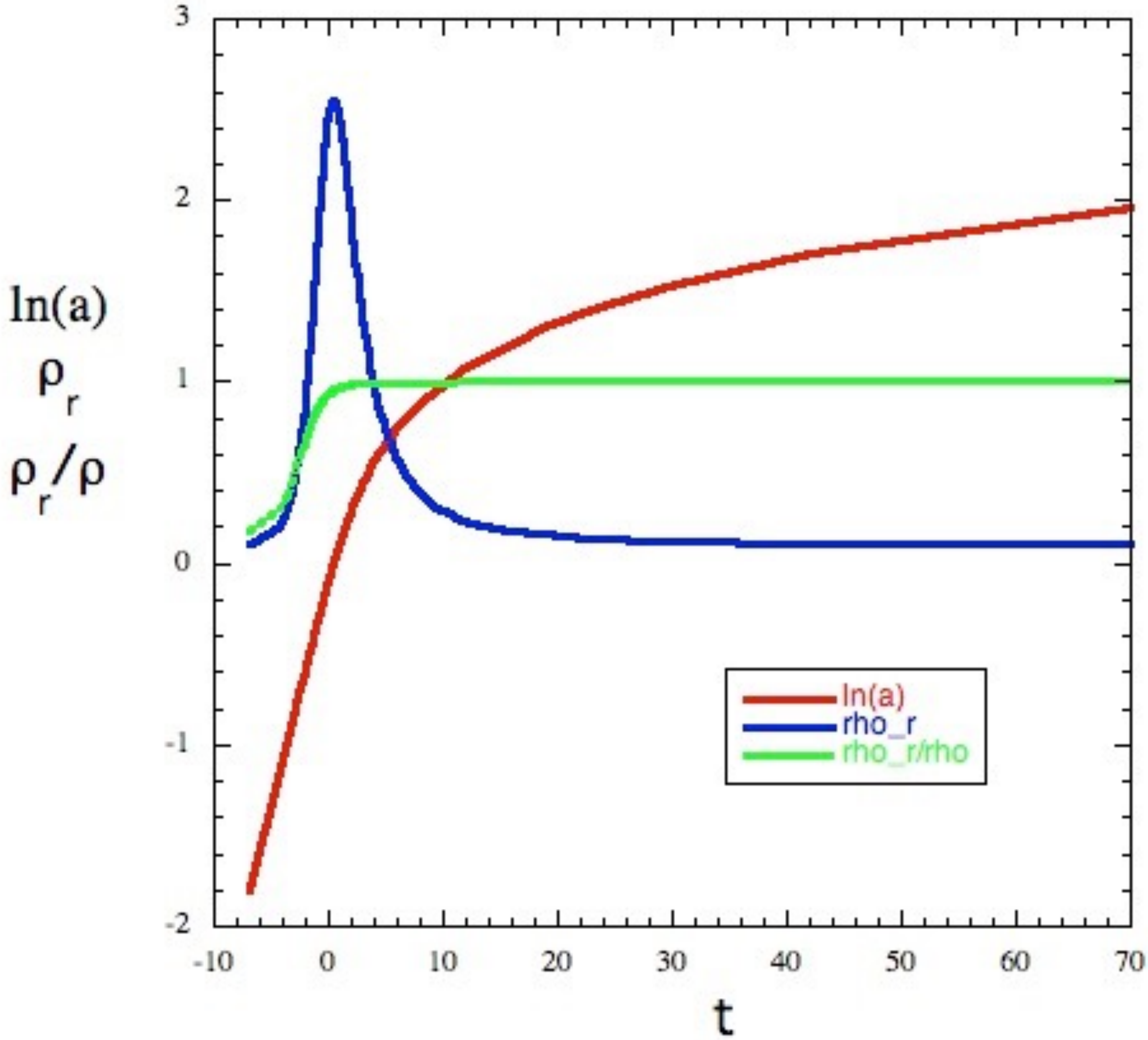
we used the fact that the inflationary density must be zero if the total density is zero.

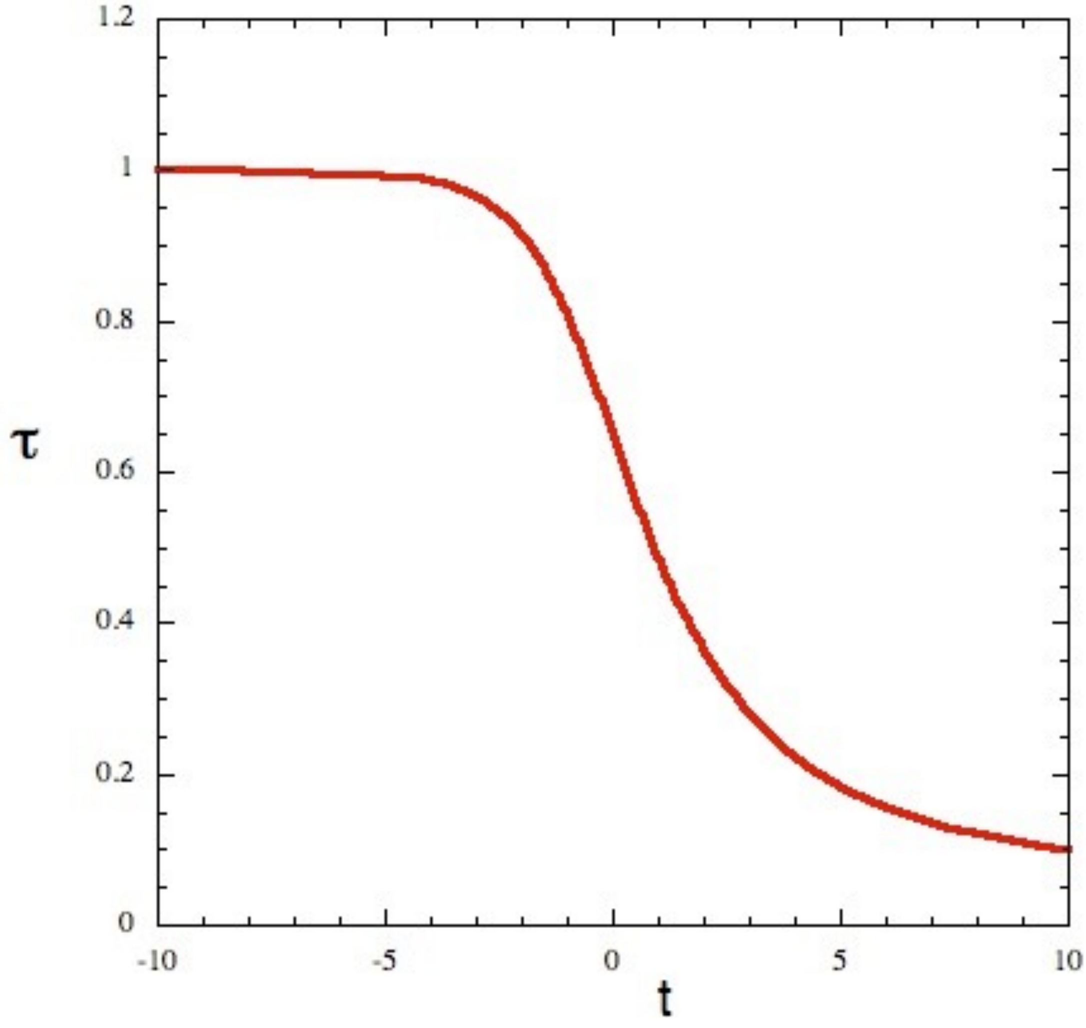
We now choose ρ_0 to be the initial inflationary density ρ_I .Then initially

$X = Y = 1$ so $\nu = 1$

$$X^2 = 2Y - Y^2$$

With this simple relationship between the total density and the inflationary density one can integrate all the equations to find how the universe evolved through the inflationary and radiation eras.


3
2
1
0
-1
-2
-10
0
10
20
30
40
50
60
70
ln(a)
ρ_r
ρ_r/ρ
t
ln(a)
rho_r
rho_r/rho



1.2
1
0.8
0.6
0.4
0.2
0
τ
-10
-5
0
5
10
t

How is this changed if the universe is closed? Near the end of inflation the curvature is very small so we treat it as a perturbation. Starting from unchanged initial conditions $X = Y = 1$ we find $X^2 = 2Y - Y^2$ is replaced by $X^2 = 2Y - Y^2 - 2\delta(\tau)$.

where $\delta(\tau) = \frac{9kc^2}{\sqrt{2}\omega^2 a_h^2}[\pi/2 - \tan^{-1}(\tau/\sqrt{1-\tau^2})]$.*

here τ is a parameter related to time that runs from one in the infinite past to 0 in the far future. So δ starts at zero and increases to a constant. a_h is the scale factor when only half of the inflationary density remains.

$$\rho_r/\rho_I = X^2 - Y^2 = 2Y - 2Y^2 - 2\delta$$

so the radiation density will vanish when

$$Y = 1, \quad \delta = 0$$
$$Y \doteq \delta(0)$$

When there is no radiation the viscous term becomes zero so the inflation density stops decaying; a tiny piece survives the radiation era and becomes the constant we have predicted.

The closure of the universe changes the conditions just a little so that dissipation ceases just before all the inflationary density has decayed; a tiny bit remains and, after all other ingredients have expanded to a very low density, this remnant of the original inflation survives as the Lambda term of Cosmology!

This tiny remnant of the original inflationary density becomes very important at late times because it does not expand away like the radiation and matter but becomes the Lambda term of Cosmology. We now have a relationship that links the current Lambda term to initial inflation it gives

$$\sqrt{\Lambda c^2} = \sqrt{8\pi G \rho_\Lambda} = \frac{(3\pi/2)kc^2}{\sqrt{16\pi\rho_I}\ \ a_h^2}$$

On this theory we see that the universe must have positive curvature. Also we can use current values, as

$$\rho_r a^4 = 2\rho_I a_h^4 [\tfrac{1-\tau^2}{1+\tau^2}]^2 \rightarrow 2\rho_I a_h^4$$

In terms of the currently popular Omegas of Cosmology our result is

$$\sqrt{\Omega_\Lambda\ \Omega_r} = -(\pi/2)\Omega_k$$

$$\Omega_k = -kc^2/(a^2 H_0^2), \qquad \Omega_\Lambda = \frac{8\pi G\rho_\Lambda}{3H_0^2}, \qquad \Omega_r = \frac{8\pi G\rho_r}{3H_0^2}$$

Planck results are

$$\Omega_\Lambda = 0.7, \quad \Omega_r = 8.8\ 10^{-5}, \quad \Omega_k = -0.005 \pm 0.017$$

our relationship gives

$$\Omega_k = -0.0050 \pm 0.0001$$

when the Planck data is supplemented with that from Baryon Acoustic Oscillations the result is

$$\Omega_k = -0.00 \pm 0.005$$

so our result will soon be disproved/accepted!

Since we know Omega_k we can find the current scale factor and from it get the current volume of the whole closed universe which is much bigger than the observable part

$$a = 14.1c/H_0, \qquad V_T = 2\pi^2 a^3 = 55777(c/H_0)^3$$

In future Baryon Acoustic Oscillations on the scale of 150 Mpc will accurately determine whether the universe is curved or flat at the Omega_k~+/-0.001 level. Probably the most accurate results will come from an broad survey of the CO line as a function of redshift with the SKA